Intrinsic l/*f* Noise in Doped Silicon Thermistors for Cryogenic Calorimeters

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ABSTRACT

We have characterized the intrinsic 1/*f* noise of ion-implanted silicon thermistors in the 0.05 − 0.5 K temperature range. This noise can have a significant effect on detector performance and needs to be taken into account in the design optimization of infrared bolometers and x-ray microcalorimeters. The noise can be reasonably well fit as $\Delta R/R$ fluctuations whose spectral density varies as 1/*f* and increases steeply with lower doping density and lower temperatures. The observed 1/*f* noise can be approximated as a resistance fluctuation: $\langle \Delta R^2 \rangle / R^2 = 0.034 (T_0 / 1\mathrm{K})^{2.453} (T_e / 0.15\mathrm{K})^{-(5.2+0.9\log(T_0))} / (3.5 \times 10^{18}\,\mathrm{cm}^{-3} V_{\mathrm{therm}}\, f)$.

Keywords: detectors, calorimeters, bolometers, x-ray, infrared, 1/f noise.

1. INTRODUCTION

Ion implanted silicon thermistors have been used successfully as thermometers for high-resolution X-ray microcalorimeters. The energy resolution obtained, however, has not generally been as high as that predicted by the thermodynamic model for a calorimeter with an ideal resistive thermometer, as given in ref. 1. In the limit of low photon energies, variable energy loss and position dependence in the absorber are usually not important. One of the remaining effects is the electrical nonlinearity of these devices, where the current increases faster than linearly with applied voltage, even when the lattice temperature is held constant. This can be interpreted as a field-dependent resistance, or as a “hot electron” effect, where the power dissipated in the device heats the electrons above the lattice temperature, and the resistance is taken to be linear, but a function of the electron temperature rather than that of the lattice. A good empirical fit to the apparent electron-lattice thermal conductivity can be obtained with a power-law dependence on T with an exponent near 5.[2,3] This nonlinearity causes a reduction in the effective sensitivity of the thermometer that limits how small its volume and heat capacity can be made, and also puts a lower limit on the thermal time constant of the detector.[3]

This still fails to account for much of the observed reduction in resolution. Implanted thermistors where the silicon lattice was firmly heat sunk showed a white noise just equal to the expected Johnson noise with no bias, but developed excess noise at low frequencies when a bias current was applied. Typical noise spectra with increasing current can be seen in figure 10 of ref. 4. The low frequency noise follows an approximately 1/*f* power spectrum, and increases with increasing bias current. At the same time, the white noise level drops to a level about consistent with calculated Johnson noise for the value of resistance observed at the bias point, which is also dropping with increasing bias power due to the nonlinearity. Other resistive systems have 1/*f* behavior that has been shown to be due to $\Delta R^2/R^2$ fluctuations that are independent of bias current.[5] In our case, the nonlinear behavior makes extracting the intrinsic noise behavior problematic, but in the end it appears to depend on only two parameters: the doping density of the thermistor and its resistivity at the bias point.

2. DATA

We have measured a number of ion-implanted silicon thermistors with T_0’s ranging from 1.5 K to 40 K over the temperature range 0.06 – 0.3 K in an adiabatic demagnetization refrigerator. In some cases measurements were extended to 0.4 K to overlap similar data that had been taken about ten years earlier at 0.26 to 0.5 K on devices with T_0’s up to 52 K. The fabrication of these thermistors is described in ref. 6. The data were all reduced as follows: a) The measured noise spectra were fit with a white noise plus a power-law component. The spectra extended from 1 Hz to 25 kHz, and at all but the lower biases, the low-frequency component dominated below 10 Hz. The fitted power law exponent was usually between

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0.9 and 1.1, but in all cases the fitted value at 5 Hz was used in subsequent analyses, which assumed 1/*f* behavior. b) The fitted ΔV^2 spectrum was converted to ΔR^2 using the value of the load resistor, the applied bias voltage, and the measured resistance (*V/I*) of the thermistor at the bias point. c) This value for ΔR^2 was corrected for the effect of the nonlinearity, which modifies the observed voltage fluctuations. In the hot electron picture, this is just the normal electrothermal feedback effect. The correction can have either sign, depending on the device resistance relative to the load resistance, and was occasionally as large as a factor of three. d) The measured resistance at the bias point was used to normalize this to $\Delta R^2/R^2$. Note that this normalization changes drastically with bias, due to the rapid drop in resistance with increasing current.

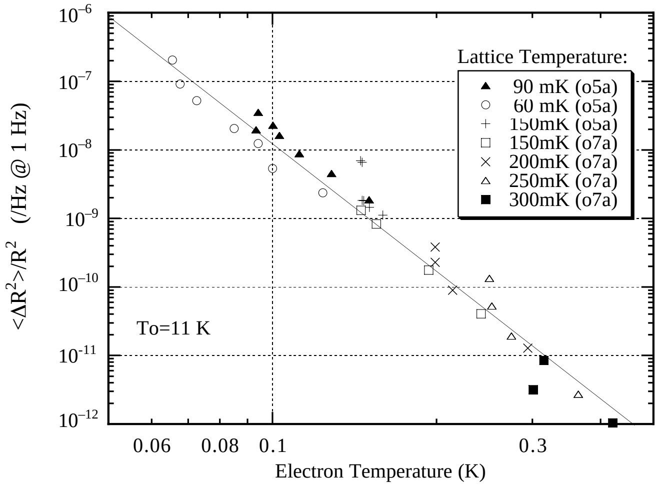

Figure 1 Resistance fluctuations as a function of the apparent thermometer temperature for several lattice temperatures. The fluctuations appear to depend only on the apparent temperature, which increases with bias power.

The relative resistance fluctuations are observed to decrease rapidly both with increasing bias and with increasing lattice temperature. The values for large bias at low lattice temperature overlap those for low bias at higher lattice temperatures. Resistance also decreases as a function of both these parameters, so we tried plotting the fluctuations as a function of the measured resistance. The result for one device is shown in figure 1, where we have plotted the derived values of $\Delta R^2/R^2$ for a wide range of biases at each of six lattice temperatures. Adopting the hot electron model, the resistance is uniquely determined by the electron temperature, and we have used the $R(T_{\mathrm{lattice}})$ curve obtained in the limit of small bias, where $T_{\mathrm{electron}} = T_{\mathrm{lattice}}$, to convert the measured resistance to electron temperature, T_e. As can be seen in the figure, it turns out that the relative resistance fluctuations at a given resistance are the same, regardless of whether the resistance was obtained at a low T_{lattice} and high bias or a higher T_{lattice} with a low bias. This seems to indicate that the excess noise is resistance fluctuations that are independent of bias current, but which depend strongly on the electron temperature (or equivalently, on the resistivity).

We next attempted to determine whether the excess noise was an intrinsic property of the thermistor, or might perhaps be due to the contacts (degenerate implants along opposite edges of the thermistor which provide the electrical connections) or to edge effects. It can readily be shown that any bulk $\Delta R/R$ effect that is uncorrelated in different parts of the thermistor must scale as 1/volume. Figure 2 compares the fluctuations on two devices on the same die. One has dimensions 100 µm x 100 µm x 0.3 µm and the other 400 µm x 400 µm x 0.3 µm, but both have the same resistance and temperature sensitivity. The fluctuations are 16 ± 1 times larger in the 100 µm x 100 µm device, as expected for a true bulk effect. This also establishes that no significant part of the resistance change is due to temperature fluctuations of the refrigerator, since this would have given equal resistance changes in both devices. Figure 3 shows another confirmation that the noise is not related to the contacts or edges. It compares the relative resistance fluctuations for two devices that have the same area, but different shapes. One is 40 µm wide by

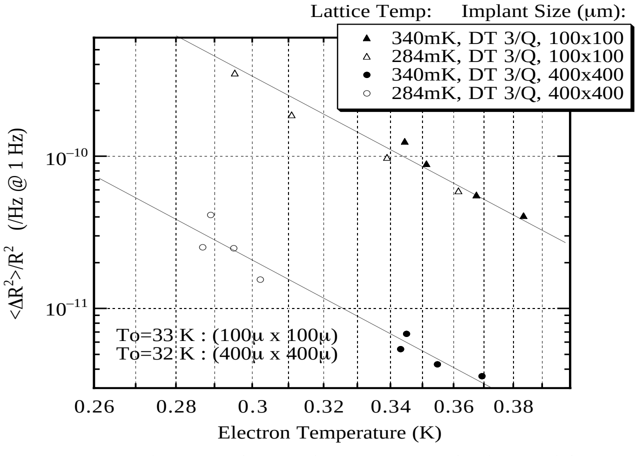

Figure 2 Resistance fluctuations for two devices with different areas, but almost identical resistance and sensitivity.

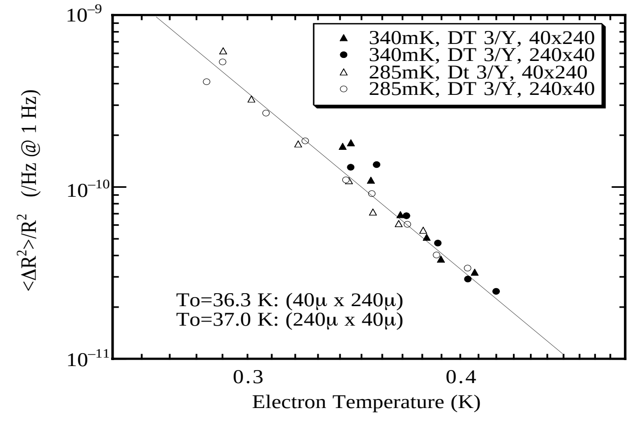

Figure 3 Resistance fluctuations vs apparent temperature for two devices with the same area, but different shapes.

240 µm long, while the other is 240 µm wide by 40 µm long, making R different by a factor of 36. The relative fluctuations are essentially identical, again indicating an intrinsic bulk effect.

To make it easy to compare different devices (and the absolute magnitude of the effect with 1/*f* noise observed in other systems), we normalize the resistance fluctuations using the Hooge alpha parameter conventional to this field. It is defined by the relation

$$\frac{\left\langle \Delta R^2 \right\rangle}{R^2} = \frac{\alpha_H}{Nf}, \tag{1}$$

where $\left\langle \Delta R^2 \right\rangle$ is the fluctuation spectral density (ohms^2/Hz), N provides the expected 1/volume dependence and is conventionally the number of charge carriers, and f is the frequency in Hz. We somewhat arbitrarily use the net number of dopant atoms as N, fixing the assumed density at 3.5 x 10^{18} cm^{-3} (the implanted densities vary by less than 30% over the entire range of devices tested). The resistance fluctuations are observed to be a much more strongly decreasing function of doping density than the minimal effect predicted by Eq. 1, so we have investigated the fluctuations as a function of the density-sensitive parameter T_0, obtained by fitting the low-bias $R(T)$ function with the behavior predicted for variable range hopping with a coulomb gap: $R(T) = R_0 e^{\sqrt{T_0/T}}$, where T_0 and to a lesser extent R_0 are functions of the doping density.[6]

Figure 4 shows some of these data, with individual power law fits for each device using data in the 0.06 – 0.3 K range. The temperature dependence is very steep, about T^{-6}, and the noise also increases rapidly with increasing T_0. As shown in figure 5, the T_0 dependence is well-fit by a single power law with slope 2.45. The fitted temperature slopes from figure 4 are plotted against T_0 in figure 6. This slope is almost constant at about –6, but there is a systematic tendency for it to be steeper at high T_0's. Adopting the straight-line fits shown in figures 5 and 6, we arrive at an expression for α_H in terms of T_0 and the electron temperature, T_e:

$$\alpha_H = .034 T_0^{2.453} \left(\frac{T_e}{0.15\mathrm{K}} \right)^{-(5.2+0.9\log(T_0))}. \tag{2}$$

Figure 7 shows the data and the fits using Eq. 2. The older data fit in well with the new, and overlap it where the new data were extended to higher temperatures. However, it is clear that the data above 0.3 K are systematically steeper than the lower-temperature fits for the same T_0.

We have also tested a small number of devices made at the NASA Goddard Space Flight Center. These have very similar specifications, but are made in a different facility according to a somewhat different recipe. The noise results are essentially identical for devices with the same T_0's. Preliminary results of tests made by the Milano group on their silicon implants give similar results, apparently with somewhat less sensitivity to T_0.[7]

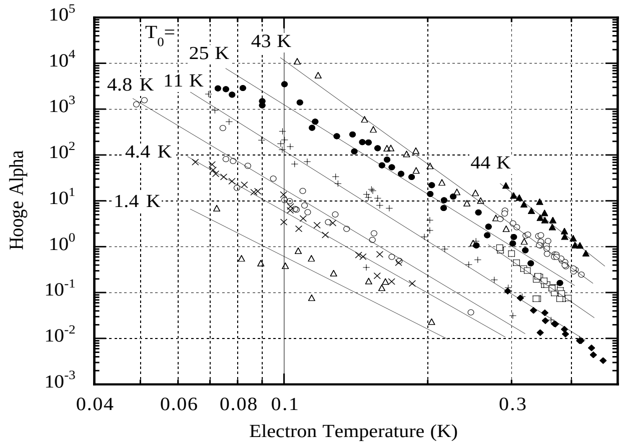

Figure 4 Excess noise parameter α_H for thermistors with different doping densities. The straight lines are the best-fit power law dependence on apparent temperature. Devices with data only above 0.26 K are from runs in a ^{3}He refrigerator in 1986.

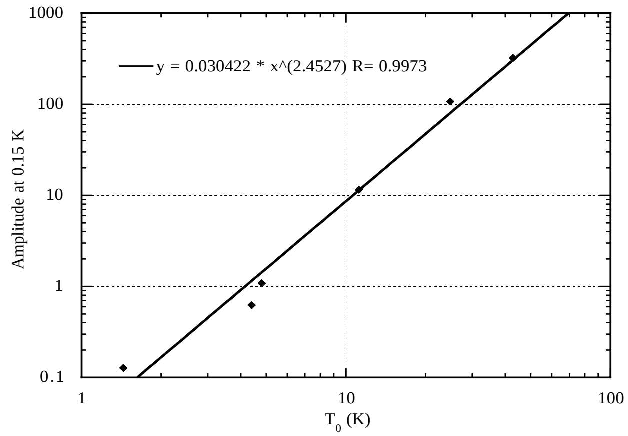

Figure 5 Power-law fit to the T_0 dependence of the amplitude of α_H at 0.15 K.

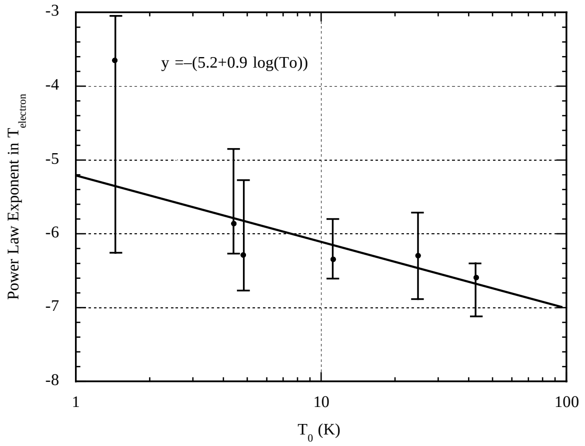

Figure 6 Fit to the T_0 dependence of the fitted power law slopes shown in figure 4.

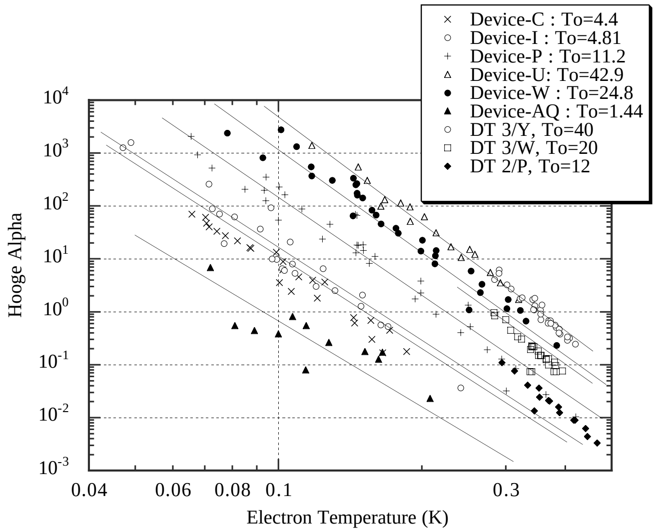

Figure 7 1/*f* noise data showing the global fits for the 0.06 – 0.2 K temperature range from Eq. 2. Above 0.3 K, the noise appears to drop faster with increasing T_e.

3. DISCUSSION

There is very little theoretical work on 1/*f* fluctuations in variable range hopping conductivity. Two papers by Shklovskii[8,9] outline a bulk process similar to the well-known McWhorter model for 1/*f* noise in MOS field effect transistors. While the general picture seems reasonable, it is difficult to make any quantitative comparisons. Our most detailed model of detector performance has always predicted better performance than actually observed, even at low energies where thermalization noise and position nonuniformities that are not included in the model have minimal effect. We can now incorporate the additional noise represented by Eq. 2 into this model, and it will be very interesting to compare these new predictions with data for existing detectors. If we can correctly account for the observed resolution, then we will have some confidence that we know how to optimize the design of a detector, and we should be able to predict the ultimate performance of detectors with this type of thermometer in any given situation (at least for low energies).

We have not completed the detailed numerical calculations, but an analytical estimate of the effect of this noise indicates that it is the dominant factor in deciding the optimum thermometer size and sensitivity for small high-resolution x-ray detectors operating below ~80 mK. The effect of the 1/*f* noise turns out to be independent of the time constant of the detector. Its effect on the resolution depends primarily on the ratio of thermometer to total heat capacity, the thermometer sensitivity (T_0), and the operating temperature.

One major question that remains about this excess noise is whether it might be a surface effect. Since all of our implants are approximately the same thickness, we cannot currently distinguish between a 1/volume and a 1/area scaling. Although Shklovskii's model describes a plausible bulk process, we have been unable to make a quantitative prediction from it, and it is possible that a surface effect could dominate. The conduction process is getting close to 2-d in these thermistors, and is becoming exponentially more so as the temperature decreases. This would offer a natural explanation for the very steep temperature dependence that is observed. It would also explain the observation that nuclear transmutation doped (NTD) germanium thermometers seem to show much less 1/*f* noise (although this is not unequivocally established: some measurements on NTD Ge seem to show very similar noise levels to the silicon implants). We are currently fabricating thicker implants with a high energy ion implanter to investigate the degree to which surface effects are important. If they turn out to dominate the noise, then it should be possible to greatly improve the ultimate performance of detectors using these thermometers simply by making them thicker.

4. REFERENCES

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