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Editor's Choice

1/f Noise and Hot Electron Effects in Variable Range Hopping Conduction

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In the course of developing microcalorimeters as detectors for astronomical X-ray spectroscopy, we have undertaken an empirical characterization of "non-ideal" effects in the doped semiconductor thermometers used with these detectors, which operate at temperatures near 50 mK. We have found three apparently independent categories of such behavior that are apparently intrinsic properties of the variable-range hopping conduction mechanism in these devices: 1/*f* fluctuations in the resistance, which seems to be a 2D effect; a departure from the ideal coulomb-gap temperature dependence of the resistance at temperatures below $T_0/24$; and an electrical nonlinearity that has the time dependence and extra noise that are quantitatively predicted by a simple hot electron model. This work has been done largely with ion-implanted Si : P : B, but similar behaviors have been observed in transmutation doped germanium.

Introduction Ion implanted silicon thermistors have been used successfully as thermometers for high-resolution X-ray microcalorimeters [1–3]. The energy resolution obtained, however, has not generally been as high as that predicted by the thermodynamic model for a calorimeter with an ideal resistive thermometer, as given in Ref. [1]. While this is partly due to imperfect thermalization and position dependence in the absorbing material, the dominant problem at least at low energies is non-ideal behavior in the thermistor. We have characterized several such behaviors, and find that they are quite systematic and predictable, allowing us to optimize the design of a detector even though we do not understand the physics behind the effects [4–6].

In this paper, we discuss three effects that have been investigated. The first of these is 1/*f* noise, which has had a significant effect on the resolution, but which apparently can be made negligible by making the implants thicker. The second is a systematic departure from the $R(T) = R_0 \exp (T_0/T)^{1/2}$ dependence expected for variable range hopping (VRH) with a Coulomb gap. This is primarily a curiosity, since it does not affect the resolution, and simply needs to be taken into account in calibration and characterization of the thermistors. The third effect is an electrical nonlinearity, or voltage-dependence of the resistance. This has a major impact on detector performance and is the primary limitation of these thermistors. There is also an additional excess noise that may be associated with the nonlinearity. The nonlinearity, its time-dependence, and the noise

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can be described quantitatively by a simple hot-electron model, even though there seems to be little theoretical justification in variable range hopping for the concept of a thermal distribution of electron energies that is independent of the phonon temperature.

1/f Noise Figure 1 shows noise spectra taken from an implanted silicon thermistor that is well-coupled to the heat sink, so that the lattice temperature does not change. If the noise excess above the Johnson noise is taken to be due to random resistance fluctuations, the relative resistance fluctuations are found to have a $1/f$ power spectrum whose magnitude depends only on the doping density and resistivity of the sample. The resistivity in turn depends on both the temperature and the power density, or electric field, due to the non-ohmic nature of these devices. As shown in Fig. 2, any combination of temperature and power density that produces the same resistivity also gives the same resistance fluctuations. Here, we have labeled the resistivity by the temperature which produces it in the limit of small power densities.

Extensive measurements of this sort are reported by Han et al. [6], who found that the $1/f$ noise power scaled accurately as 1/(number of impurity atoms) and was independent of the length to width ratio, indicating that it is an intrinsic property of the thermistor and is not related to the contacts. The Hooge alpha parameter ranged from 10^{-3} to 10^{4} over the range of doping densities and temperatures investigated. It increased with decreasing doping density, and very rapidly ($\sim T^{-7}$) with decreasing temperature. The magnitude and behavior of the noise was similar for devices made in three different facilities according to somewhat different recipes.

The thickness of all of these implanted devices was near 2300 Å, since this is the maximum easily made by a standard industrial ion implanter. This is also about the calculated radius of the critical percolation networks at the lowest temperatures investigated, so it appeared plausible that the extremely steep temperature dependence of the

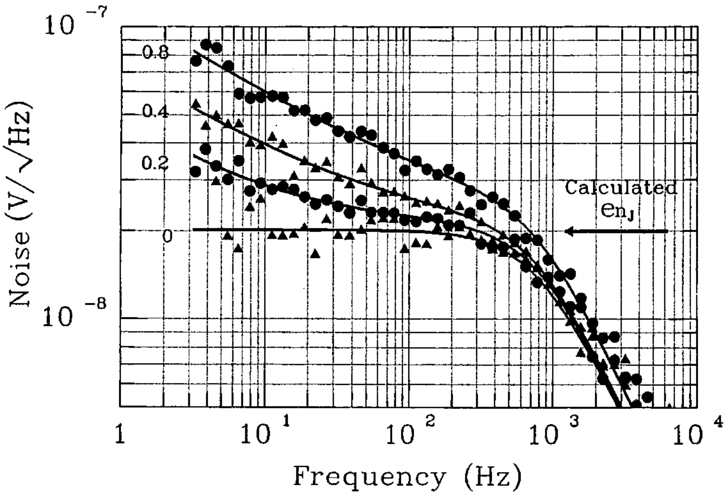

Fig. 1. Noise spectra of an ion-implanted silicon thermistor strongly coupled to a heat sink at 60 mK. Numbers show bias current in nA. With no bias, the device shows only expected Johnson noise. Substantial $1/f$ noise is apparent under increasing bias. These standard implants are about 2300 Å thick

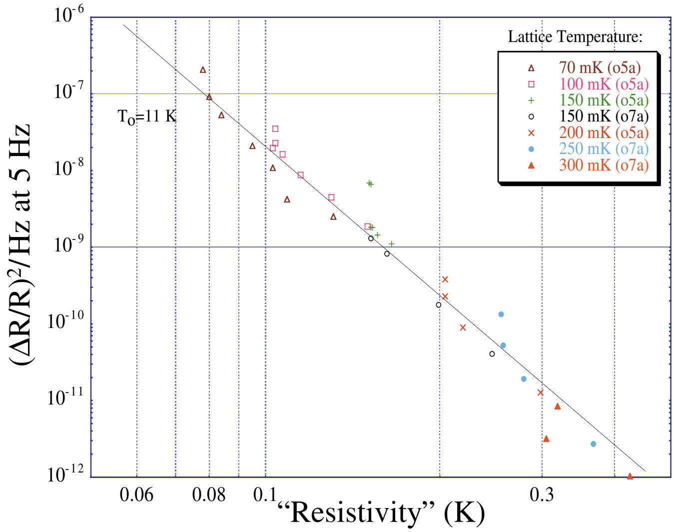

Fig. 2. Relative resistance fluctuations at 5 Hz derived from noise spectra, plotted as a function of resistivity. A given resistivity can be achieved with a high temperature and small bias, or a lower temperature with a higher bias, due to the voltage-dependence of the resistance. The resistance fluctuations are observed to be the same at a given resistivity independent of the combination of temperature and bias used to achieve it. Resistivity is given in terms of temperature in the limit of small bias

$1/f$ noise was due to the onset of two-dimensional effects. We have recently constructed thicker devices, which combine ion implantation with thermal diffusion to uniformly distribute the dopants over the 15000 Å thickness of a silicon-on-insulator layer bounded on both sides by silicon oxide. Figure 3 shows that the $1/f$ noise is greatly reduced from what would be observed in a thinner device with the same total volume. In practical terms, this should result in about a factor of two improvement in energy resolution for microcalorimeters employing these thermometers.

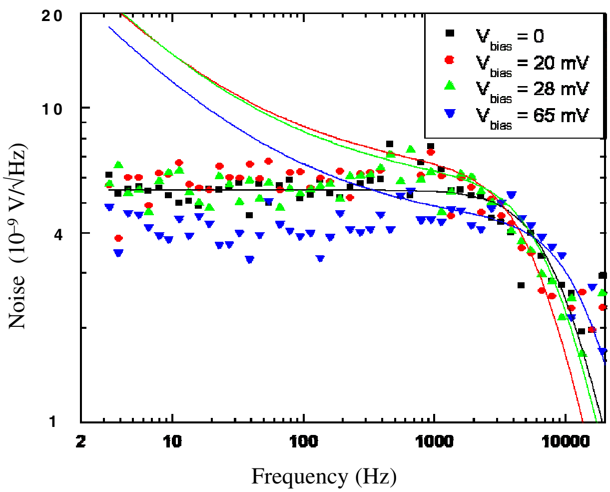

Fig. 3. Noise spectra from a device that is 15000 Å thick. Solid lines show noise expected from fits to the thinner implants after normalization to the same volume

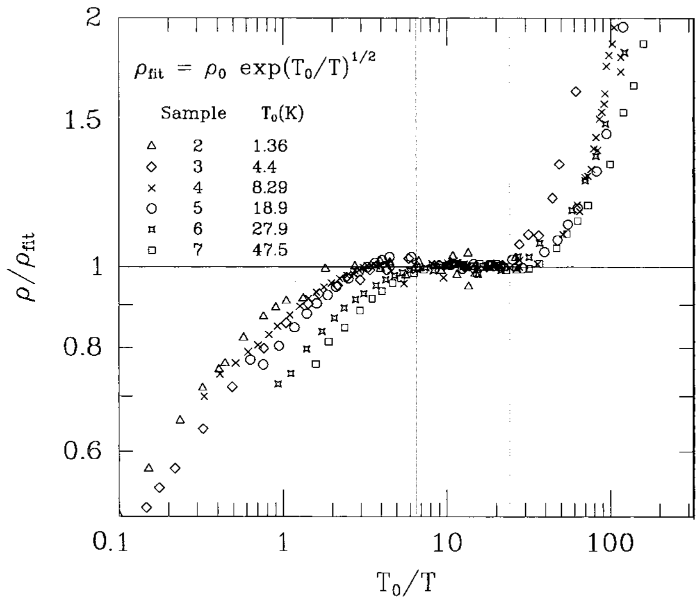

Fig. 4. Measured resistance divided by fits to the $R(T) = R_0 \exp(T_0/T)^{1/2}$ dependence expected for variable range hopping with a coulomb gap, plotted against reduced reciprocal temperature for samples with various doping densities. The downward deviations at high temperature are probably an artifact produced by the low-density wings of the implant profile

Deviations from Coulomb Gap *R*(*T*) Figure 4 shows that at low temperatures, the resistance rises sharply above the function expected for VRH with a coulomb gap, starting at about $T_0/24$. At higher temperatures the standard function fits to a few percent over more than two orders of magnitude in resistance. Adjusting the 0.5 exponent or introducing a T^S prefactor do not help the situation appreciably. We originally thought that this might also be a 2D effect [4], but the thicker implants show identical behavior. Figure 5 shows that $R(T)$ for a thick NTD germanium device also has exactly the same form. One likely explanation is the formation of a magnetic hard gap, as proposed by Shlimak [7].

Electrical Nonlinearity A dependence of resistance on electric field is expected in hopping conduction. However, an extensive investigation of the effect in doped Si and Ge showed that the expected exponential form is observed only for low temperatures and doping densities, and a different functional form is needed over most of the range of interest for microcalorimeters [5]. A physical interpretation of the simple function used to fit the data here is inspired by the well-established hot electron effect in metals, as shown in Fig. 6. The resistance is assumed to be a function of the electron temperature only, while bias power is dissipated in the electron system and must be conducted to the lattice through a thermal resistance with a power-law temperature dependence. Since the $R(T_e)$ function can be measured directly in the low-power limit, this model has only two free parameters, which are the G_0 and β of the electron-phonon "thermal conductance". The results of such a fit over a wide range of lattice temperature and power are shown in Fig. 7. The inset shows the rather precise power-law behavior of

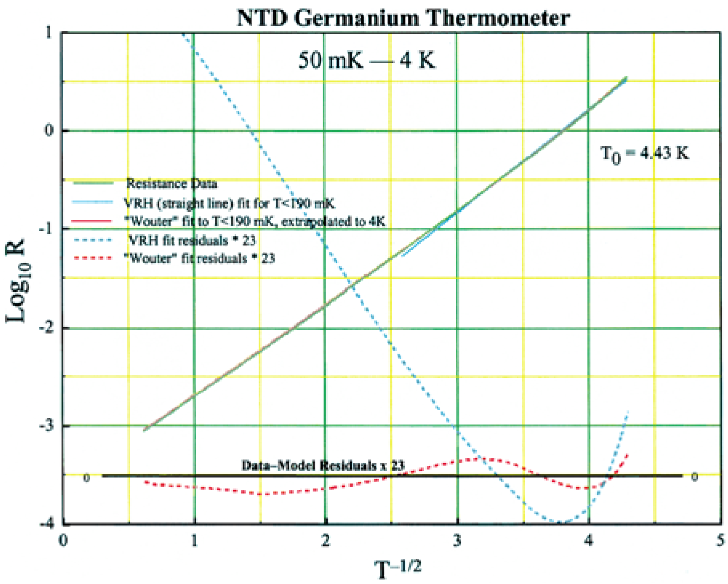

Fig. 5. Resistance vs. temperature for an NTD germanium thermometer ($0.2 \times 1 \times 2$ mm^3). A straight-line "coulomb gap" model has been fit to temperatures below 190 mK, as has a function that fits the deviations shown in Fig. 4 for thin silicon implants. The latter fit has no additional free parameters. It has been extrapolated to 4 K, but is indistinguishable from the data line on this plot. The residuals from both models are plotted on an expanded scale at the bottom. Maximum deviations of the function derived from the silicon implants are ±2% over four decades of resistance

the apparent thermal conductivity. The exponent β is constant with temperature, but changes from about 4.3 to 5.8 as the doping density is varied over a wide range [5].

One would be inclined to regard the agreement with the functional form predicted by a hot electron model as a convenient accident, since there is no physical picture that we know of for establishing a thermal spectrum of occupied electron energy states at anything but the phonon temperature. Applied fields can raise the mean energy of the electrons, but it is not clear how they could attain a thermal distribution among themselves. If we wish to press the point, however, the model of Fig. 6 makes additional

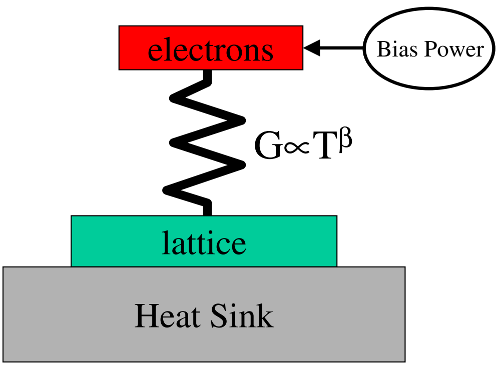

Fig. 6. "Hot electron" model for observed nonlinearity in doped semiconductor thermistors. In analogy with hot electron effect in metals, the electrons are assumed to establish a temperature independent of the phonon temperature of the lattice, and the resistance depends only on this "electron temperature"

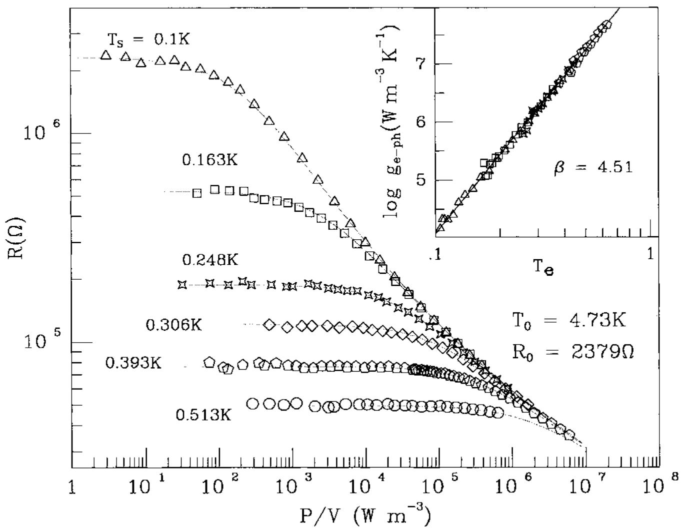

Fig. 7. Nonlinearity data fit to the hot electron model. With only two free parameters, good fits are obtained over a wide range of lattice temperature and power density

predictions when it is assumed that the electron system has a finite heat capacity. In such a case, there will be a characteristic time $\tau \equiv C_e/G_{e-ph}$, and standard bolometer theory [8] predicts a frequency dependence of the device impedance as shown in Fig. 8. The electrical nonlinearity is represented by the I–V curve shown at the left. At low frequencies, the impedance Z must simply follow the local slope of this curve. However, for $\omega \gg 1/\tau$, the temperature and therefore the resistance will not change appreciably, and $Z = R$ ($\equiv V/I$). At intermediate frequencies, there will be a phase shift, and the impedance should follow a semi-circle in the complex plane, as shown at the right.

We have measured $R(T_{SINK}, P)$ and $Z(\omega)$ for a number of devices with the Si lattice cemented directly to a heat sink, as in Fig. 6. We find good fits to this "hot electron"

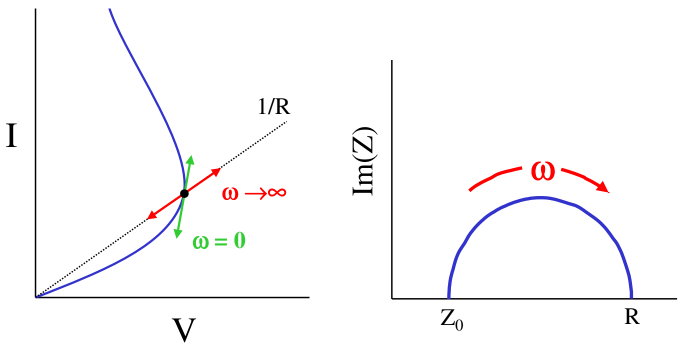

Fig. 8. If there is a heat capacity associated with the hot electrons, there should be an associated thermal time constant $\tau = C_{electron}/G_{electron-phonon}$. Standard thermal detector theory then predicts that the measured impedance should be a function of frequency, going from a minimum real value Z_0 at low frequencies to a value equal to the resistance at high frequencies. At intermediate values, $Z(\omega)$ will trace a semicircle on the complex plane

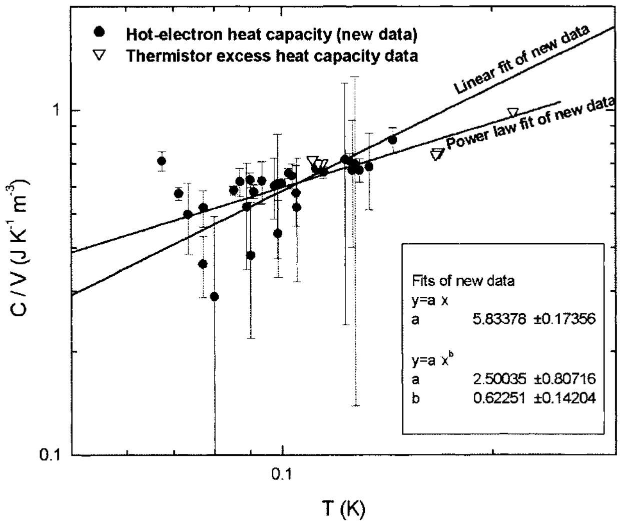

Fig. 9. Inferred electronic heat capacity as a function of electron temperature obtained by multiplying the time constants determined from $Z(\omega)$ by $G_{\text{electron-phonon}}$ as obtained from fits to $R(T, P)$. The triangles show the total excess heat capacity of the implanted material from direct measurements

model for both, and can use the fitted values of τ and G_{e-ph} to find the apparent values of C_e. These are plotted against the derived T_e in Fig. 9. The apparent values of C_e show a temperature variation that appears closer to $T^{0.6}$ than T (the two straight line fits on the figure), but they are in good agreement with the conventional measurements of the total heat capacity of the doped material as shown by the open triangles. This total should be an upper limit to what is measured as C_e, but it is not obvious that one would expect the entire heat capacity of the impurity system to be coupled to the conducting electrons, as seems to be the case.

Pressing the hot electron picture yet further, we would expect fluctuations in the energy content of the electron system due to random transport of energy between it and the lattice. This is entirely equivalent to the well-known thermodynamic fluctuation noise of conventional bolometers, and the magnitude of the transported power density spectrum should be $\sqrt{4k_B T^2 G}$ W/$\sqrt{\text{Hz}}$, with small corrections for the steady-state temperature difference between the system and heat sink [8]. The relative temperature fluctuations depend only on G in the low frequency limit, and G has been determined from the observed $R(T_{\text{SINK}}, P)$. The transduced response to these temperature fluctuations can be calculated from standard bolometer theory [8], and results in a white spectrum in the limit of low frequencies. Noise measurements made on glued-down samples show a white noise component in excess of the Johnson noise. This excess is plotted in Fig. 10 as a function of the calculated noise due to these thermodynamic fluctuations. Once again, there is rather good agreement with the naïve predictions of the hot electron model.

We do not pretend to comment on the physics behind these observations. For our purposes, the phenomenological success of the model provides a computationally simple and convenient way to optimize the design of microcalorimeters employing these thermometers and to predict their potential performance over a wide range of conditions. With the reduction of $1/f$ noise to negligible levels, the hot electron model now seems to provide a complete and quantitative description of all observed behaviors of these devices. It is intriguing, however, that whatever the real physics underlying these

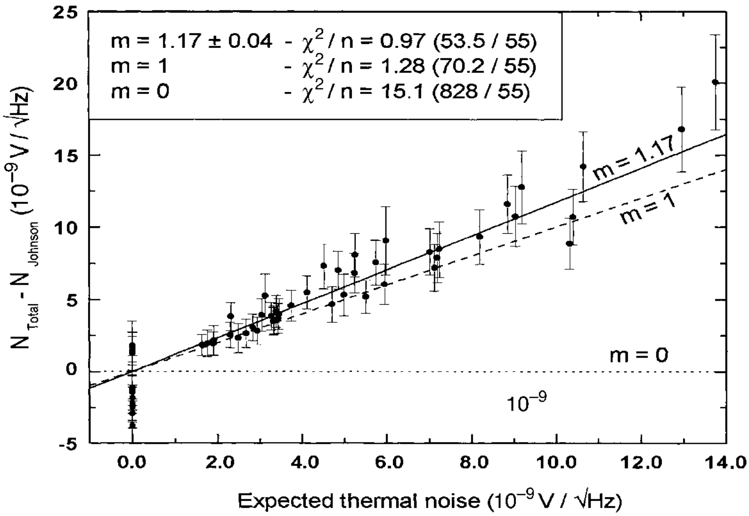

Fig. 10. Excess white noise observed in 1.5 μm thick silicon implants plotted as a function of the thermal fluctuation noise expected between the lattice and electron systems in a literal interpretation of the "hot electron" model

behaviors, it must be sufficiently related to thermal equilibrium that it can produce the same characteristic times and fluctuation statistics, in addition to the proper form of the nonlinearity.

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